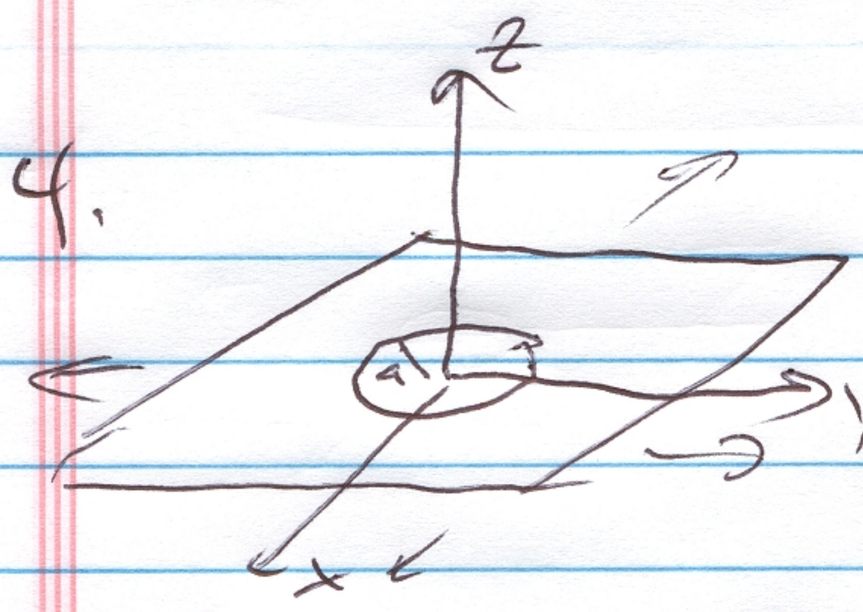
4.



a)

For full sheet with uniform σ, $z > 0$

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{z}$$

For disc with uniform $-\sigma$

$$\vec{E}_2 = -\frac{\sigma}{2\varepsilon_0}\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]\hat{z}$$

Superpose to find net field on z axis

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\frac{z}{\sqrt{z^2+a^2}} \qquad z > 0$$

b) $\vec{E}$ goes to maximum value of $\frac{\sigma}{2\varepsilon_0}$ as $z \to \infty$.

c) $\vec{E} = \frac{1}{2}\frac{\sigma}{2\varepsilon_0}\hat{z} = \frac{\pi}{4\varepsilon_0}\hat{z} = \frac{\sigma}{2\varepsilon_0}\frac{z}{\sqrt{z^2+a^2}}$

$$\frac{1}{2} = \frac{z}{\sqrt{z^2+a^2}} \qquad \frac{1}{4}z^2 + \frac{1}{4}a^2 = z^2$$

$$3z^2 = a^2 \qquad z = \frac{1}{\sqrt{3}}a$$

for 1/2 max value.