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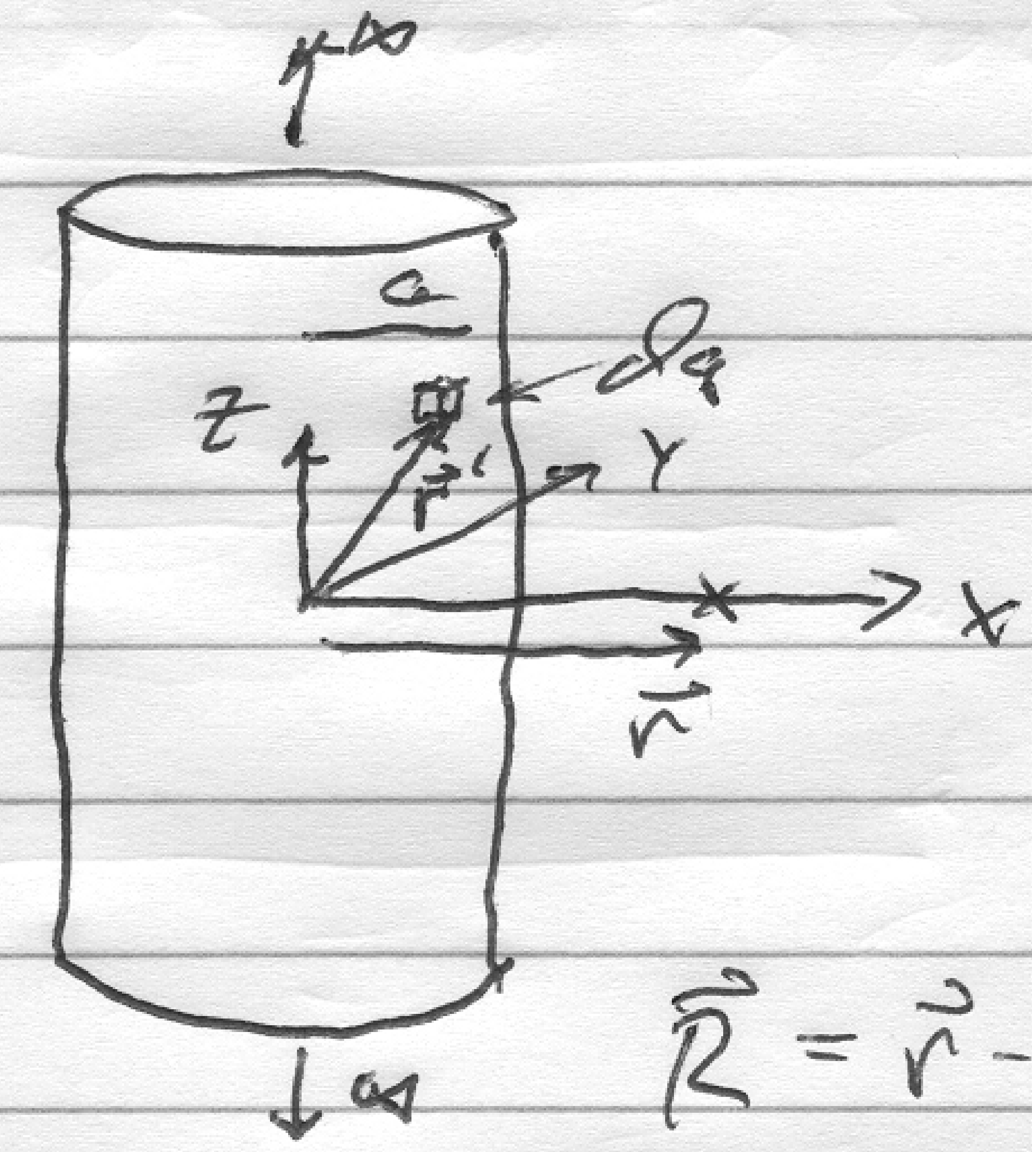

Symmetry says no loss of generality by using $\vec{r} = x\hat{x}$ and that field will have radial component only

$$\vec{R} = \vec{r} - \vec{r}' = x\hat{x} - \rho'\hat{\rho}' - z'\hat{z}$$

$$= (x - \rho'\cos\varphi')\hat{x} - \rho'\sin\varphi'\hat{y} - z'\hat{z}$$

$$R = \left((x-\rho'\cos\varphi')^2 + \rho'\sin^2\varphi' + z'^2\right)^{1/2} = \left[x^2 + (z')^2 - 2x\rho'\cos\varphi' + (\rho')^2\right]^{1/2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\vec{R}}{R^3}\,dq = \frac{\rho_{ch}}{4\pi\varepsilon_0}\int_{V'}\frac{\vec{R}}{R^3}\rho'\,d\rho'\,d\varphi'\,dz'$$

$$= \frac{\rho_{ch}}{4\pi\varepsilon_0}\int_0^a\int_0^{2\pi}\int_{-\infty}^{\infty}\frac{\left[(x-\rho'\cos\varphi')\hat{x} - \rho'\sin\varphi'\hat{y} - z'\hat{z}\right]dz'\,d\varphi'\,\rho'\,d\rho'}{\left(x^2+(z')^2 - 2x\rho'\cos\varphi' + (\rho')^2\right)^{3/2}}$$

Integrate over z': $\hat{z}$ integrand odd $\to 0$

$$\int_{-\infty}^{\infty}\frac{dz'}{(\alpha^2+(z')^2)^{3/2}} = \frac{1}{\alpha^2}\left.\frac{z'}{\sqrt{(z')^2+\alpha^2}}\right|_{-\infty}^{\infty} = \frac{2}{\alpha^2}$$

now

$$\vec{E} = \frac{\rho_{ch}}{2\pi\varepsilon_0}\int_0^a\int_0^{2\pi}\frac{(x-\rho'\cos\varphi')\hat{x}\,d\varphi'\,\rho'\,d\rho'}{x^2+(\rho')^2 - 2x\rho'\cos\varphi'}$$

($\hat{y}$ component dies by symmetry)

eqn 3-16, take $A = x$, $B = \rho'$, note that $\int_0^{2\pi} = 2\int_0^{\pi}$ since symmetric about π

$$\int_0^{2\pi}\frac{(x-\rho'\cos\varphi')\hat{x}\,d\varphi'}{x^2+(\rho')^2-2x\rho'\cos\varphi'} = \begin{cases} 2\pi/A \;(x) & \text{if } x^2 > (\rho')^2 \\ 0 & \text{if } x^2 < (\rho')^2\end{cases}$$

$x^2 > a^2$ always outside

$$\vec{E} = \frac{\rho_{ch}}{2\pi\varepsilon_0}\frac{2\pi}{x}\int_0^a\rho'\,d\rho'\,\hat{x} = \frac{\rho_{ch}a^2}{2\varepsilon_0 x}\hat{x}$$

$x < a$ inside

φ' integral $= 0$ for $x < \rho'$, $= \frac{2\pi}{x}$ for $x > \rho'$

So

$$\vec{E} = \frac{\rho_{ch}}{2\pi\varepsilon_0}\left[\int_0^x\rho'\,d\rho' + \int_x^a(0)\rho'\,d\rho'\right]\hat{x} = \frac{\rho_{ch}x}{2\varepsilon_0}\hat{x}$$