3305 HW #3 Solutions

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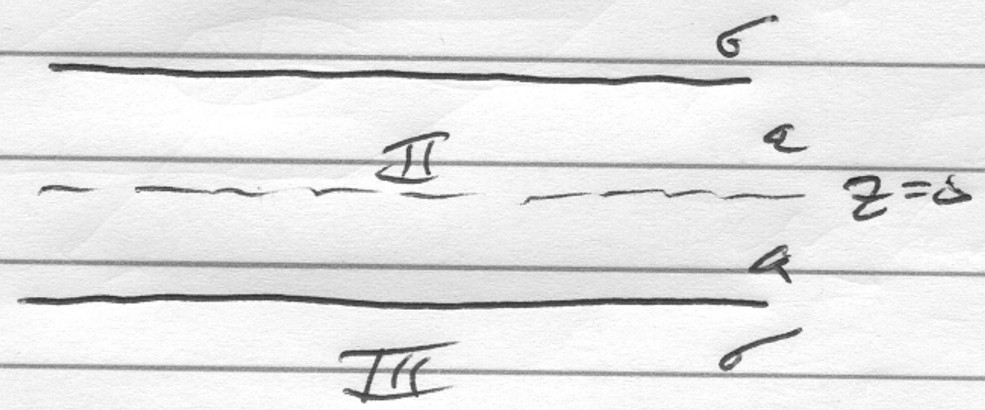

Superposition!

in region I $\vec{E} = \vec{E}_{top} + \vec{E}_{bottom} = \left(\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)\hat{z} = \underline{\frac{\sigma}{\varepsilon_0}\hat{z}}$

" " II $\vec{E} = \left(-\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)\hat{z} = 0$

in region III $\vec{E} = \left(-\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\right)\hat{z} = \underline{-\frac{\sigma}{\varepsilon_0}\hat{z}}$

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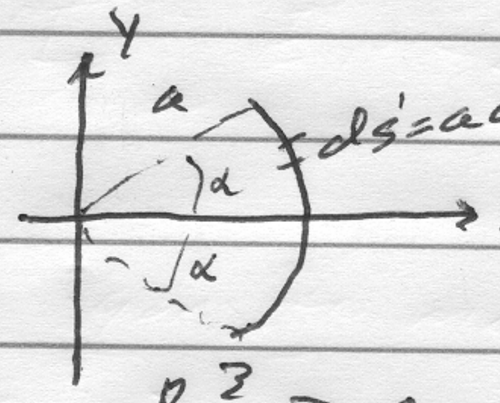

Uniform λ, want $\vec{E}$ at $\vec{r} = z\hat{z}$

$\vec{r}' = a\hat{\rho}'$ so $\vec{R} = z\hat{z} - a\hat{\rho}'$

or $\vec{R} = -a\cos\varphi'\hat{x} - a\sin\varphi'\hat{y} + z\hat{z}$

$R^3 = \text{constant} = (z^2 + a^2)^{3/2}$

and $dq = \lambda\, ds' = \lambda a\, d\varphi'$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq\,\vec{R}}{R^3} = \frac{\lambda a}{4\pi\varepsilon_0}\int_{-\alpha}^{\alpha} \frac{[-a\cos\varphi'\hat{x} - a\sin\varphi'\hat{y} + z\hat{z}]}{(a^2+z^2)^{3/2}}\, d\varphi'$$

$\hat{y}$ integral $\to 0$, odd integrand over even limits

So

$$\vec{E} = -\frac{\lambda a^2}{4\pi\varepsilon_0(a^2+z^2)^{3/2}}\sin\varphi'\Big|_{-\alpha}^{\alpha}\hat{x} + \frac{\lambda a z}{4\pi\varepsilon_0(a^2+z^2)^{3/2}}\varphi'\Big|_{-\alpha}^{\alpha}\hat{z}$$

$$\underline{\vec{E} = \frac{-\lambda a^2 \sin\alpha}{2\pi\varepsilon_0(a^2+z^2)}\hat{x} + \frac{\lambda a z \alpha}{2\pi\varepsilon_0(a^2+z^2)^{3/2}}\hat{z}}$$

For a complete circle $\alpha \to \pi$ $\sin\pi = 0$

and $\alpha \to \pi$, so

$$\vec{E} = \frac{\lambda a z}{2\varepsilon_0(z^2+a^2)^{3/2}}\hat{z} \quad \text{as it must!}$$