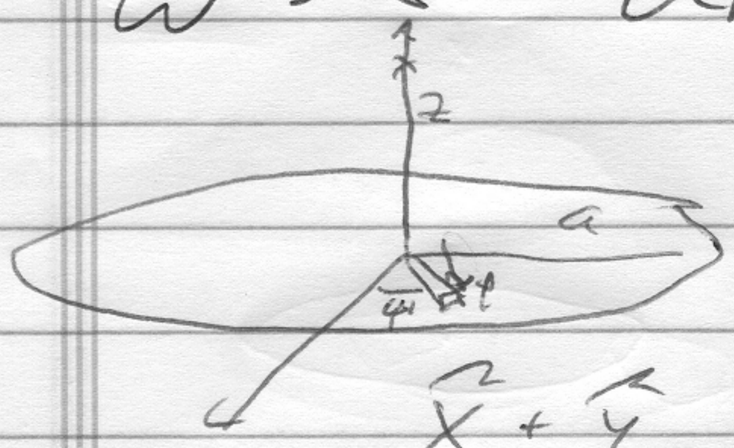
(W) 3-6 Uniform plane sheet (infinite kind)



$\vec{r} = z\hat{z}$ $\vec{r}' = \rho'\hat{\rho}'$ $a \to \infty$

$\vec{R} = z\hat{z} - \rho'\hat{\rho}' = z\hat{z} - \rho'\cos\phi'\hat{x} - \rho'\sin\phi'\hat{y}$

$R^3 = (z^2+\rho^2)^{3/2}$ $dq = \sigma\, \rho\, d\rho\, d\phi$

$\hat{x} + \hat{y}$ components die by symmetry

$$\vec{E} = k\int_S \frac{dq}{R^3}\,\vec{R}\cdot\hat{z}\,\hat{z} = k\sigma z \int_0^{2\pi}\int_0^{\infty} \frac{\rho\, d\rho\, d\phi}{(z^2+\rho^2)^{3/2}}\,\hat{z}$$

$$= 2\pi k\sigma z \int_0^{\infty} \frac{\rho\, d\rho}{(z^2+\rho^2)^{3/2}}\,\hat{z}$$

$$= 2\pi k\sigma z \left[-\frac{1}{\sqrt{z^2+\rho^2}}\right]\Big|_0^{\infty}\,\hat{z}$$

$$= 2\pi k\sigma z \left[0 - \left(-\frac{1}{|z|}\right)\right]\hat{z}$$

$$\vec{E} = 2\pi k\sigma \frac{z}{|z|}\,\hat{z}$$ in terms of ε_0

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\,\hat{z}$$