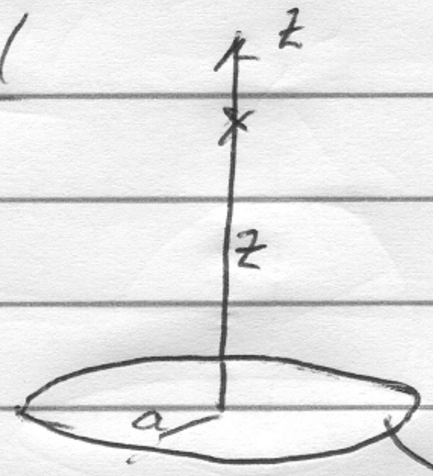
2-11



$\sigma = A\rho^2$

Units of A

$A = \frac{C}{\rho^2}$ $\quad \frac{C}{m^4}$

Divide into ringlets of radius ρ and area

$da = 2\pi\rho\, d\rho$

$$Q_{tot} = \int_0^a \sigma\, da = \int_0^a A\rho^2\, 2\pi\rho\, d\rho = 2\pi A \int_0^a \rho^3 d\rho$$

$$Q_{tot} = \frac{\pi A}{2} a^4$$

for $\vec{E}$, use ring result for $d\vec{E}(\rho)$

ring $\vec{E} = \frac{KQx}{(x^2+a^2)^{3/2}}\hat{z}$ $\quad Q \to dq = \sigma\, da$, $x \to z$, $a \to \rho$

so

$$d\vec{E} = K\,\frac{2A\rho^2\, 2\pi\rho\, d\rho}{(z^2+\rho^2)^{3/2}}\hat{z} = 2\pi AKz\,\frac{\rho^3 d\rho}{(z^2+\rho^3)^{3/2}}\hat{z}$$

$$\int \frac{x^3 dx}{(a^2+x^2)^{3/2}} = \sqrt{a^2+x^2} + \frac{a^2}{\sqrt{a^2+x^2}}$$ tables

$$\vec{E} = 2\pi AKz \int_0^a \frac{\rho^3 d\rho}{(z^2+\rho^2)^{3/2}}\hat{z}$$

$$= 2\pi AKz\left[\sqrt{z^2+\rho^2} + \frac{z^2}{\sqrt{z^2+\rho^2}}\right]_0^a \hat{z}$$

$$= 2\pi AKz\left[\sqrt{z^2+a^2} + \frac{z^2}{\sqrt{z^2+a^2}} - 2z\right]\hat{z}$$

$z \gg a$: taking $a=0$ gives zero, need next terms

$$\vec{E} = 2\pi AKz\left[z\left(1+\tfrac{1}{2}\tfrac{a^2}{z^2} - \tfrac{1}{8}\tfrac{a^4}{z^4}\right) + z\left(1-\tfrac{1}{2}\tfrac{a^2}{z^2}+\tfrac{3}{8}\tfrac{a^4}{z^4}\right) - 2z\right]\hat{z}$$

$$\approx 2\pi AKz^2\left[1+\tfrac{1}{2}\tfrac{a^2}{z^2} - \tfrac{1}{8}\tfrac{a^4}{z^4} + 1 - \tfrac{1}{2}\tfrac{a^2}{z^2} + \tfrac{3}{8}\tfrac{a^4}{z^4} - 2z\right]\hat{z}$$

$$\approx 2\pi AKz^2\left[\tfrac{1}{4}\tfrac{a^4}{z^4}\right]\hat{z}$$

$$= \frac{\pi}{2} AKa^4 \frac{1}{z^2}\hat{z}$$ but $\frac{\pi}{2}Aa^4 = Q_{tot}$

so $\vec{E} \approx \frac{KQ_{tot}}{z^2}\hat{z}$ YAY!