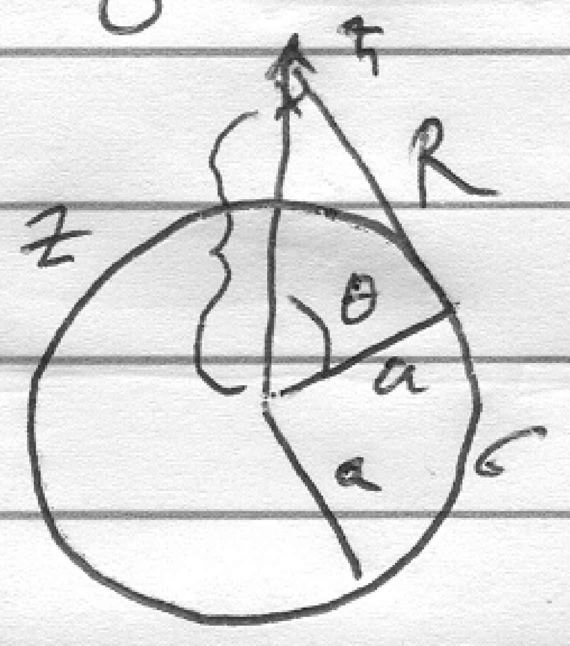
W 2-8



$$R = \sqrt{z^2 + a^2 - 2az\cos\theta'}$$

$$\vec{R} = \vec{r} - \vec{r}' = z\hat{z} - a\hat{r}'$$

$$dq = \sigma\, da = \sigma a^2 \sin\theta' d\varphi' d\theta'$$

$$\vec{E} = K \int_{S'} \frac{\vec{R}}{R^3} dq$$

$$= K \iint \frac{[(z - a\cos\theta')\hat{z} + (\sin\theta' \cos\varphi' a)\hat{x} + (a \sin\theta' \sin\varphi')\hat{y}]}{(z^2 + a^2 - 2az\cos\theta')^{3/2}} \times \sigma a^2 \sin\theta' d\theta' d\varphi'$$

φ' integral kills $\hat{x}, \hat{y}$ terms

$$\vec{E} = K\sigma a^2 \int_0^{2\pi}\int_0^{\pi} \frac{(z - a\cos\theta')\sin\theta' d\theta' d\varphi'}{(z^2 + a^2 - 2az\cos\theta')^{3/2}} \hat{z}$$

set $u = \cos\theta'$ $\quad du = -\sin\theta' d\theta'$

$\varphi' \int \to 2\pi$

$$\vec{E} = -K\sigma a^2 2\pi \int_1^{-1} \frac{(z - au)\, du}{(z^2 + a^2 - 2azu)^{3/2}} \hat{z}$$

from eqn 2-23, the u integral is

$$I = -\frac{zu - a}{z^2 (z^2 + a^2 - 2azu)^{1/2}} = \frac{1}{z^2}\left(\frac{z-a}{|z-a|} + \frac{z+a}{|z+a|}\right)$$

for $z > a$ $\quad I = -2/z^2$

for $z < a$ $\quad I = -\frac{a-z}{a-z} + \frac{z+a}{z+a} = 0$

So $\underline{z > a}$

$$\vec{E} = \frac{(2\pi) 2 K\sigma a^2}{z^2} \hat{z} = \frac{4\pi K \sigma a^2}{z^2} \hat{z}$$

$$\vec{E} = \frac{\sigma a^2}{\varepsilon_0 z^2} \hat{z} \qquad Q = 4\pi a^2 \sigma$$

So $\quad \underline{\vec{E} = \frac{Q}{4\pi\varepsilon_0 z^2} \hat{z}} \quad$ pt. charge

$z < a \quad \underline{\vec{E} = 0} \;!$