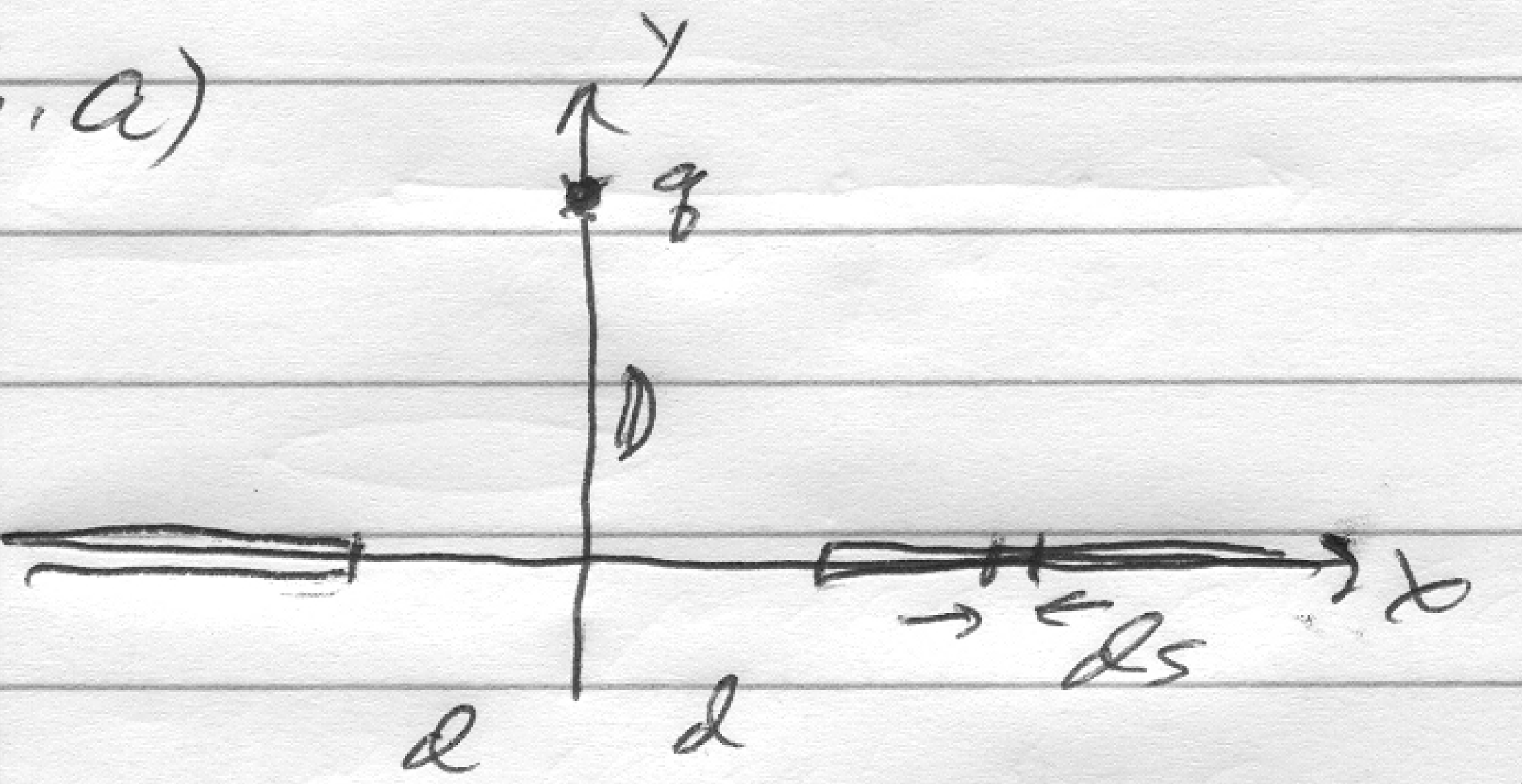
3. a)




$$ds = dx$$
$$dq = \lambda\, dx$$
$$\vec{R} = \vec{r} - \vec{r}\,' = x\hat{x} - y\hat{y}$$
$$|\vec{R}| = \sqrt{x^2+y^2}$$

$$\vec{E} = 2k\lambda \int_d^\infty \frac{\vec{R}}{R^3}\,dx = 2k\lambda\left[\int_d^\infty \frac{x\hat{x}\,dx}{(x^2+y^2)^{3/2}} + \int_d^\infty \frac{(-y)\hat{y}}{(x^2+y^2)^{3/2}}\,dx\right]$$

symmetry! vanishes by symmetry

$$\vec{E} = -2k\lambda y \int_d^\infty \frac{dx}{(x^2+y^2)^{3/2}}\,\hat{y}$$

$$\int_d^\infty \frac{dx}{(x^2+y^2)^{3/2}} = \frac{1}{y^2}\frac{x}{\sqrt{x^2+y^2}}\Bigg|_d^\infty = \frac{d}{y^2\sqrt{y^2+d^2}} - \frac{1}{y^2}$$

so

$$\vec{E} = 2k\lambda y\,\frac{1}{y^2}\left[1 - \frac{d}{\sqrt{y^2+d^2}}\right]\hat{y} \qquad y \to D$$

$$\underline{\vec{E} = \frac{2k\lambda}{D}\left[1 - \frac{d}{\sqrt{D^2+d^2}}\right]\hat{y}} \qquad \underline{\vec{F} = q\vec{E}}$$

b) $d \to 0$ $\quad \vec{E} = \frac{2k\lambda}{D}\hat{y}$

$\infty$ line charge result!

c) $D \ll d$

$$\frac{1}{\sqrt{D^2+d^2}} = \frac{1}{d\sqrt{1+D^2/d^2}} \simeq \frac{1}{d}\left(1 - \frac{1}{2}\frac{D^2}{d^2}\right)$$

So

$$\vec{E} \simeq \frac{2k\lambda}{D}\left[1 - 1 + \frac{1}{2}\frac{D^2}{d^2}\right]\hat{y}$$

$$= \frac{k\lambda}{d^2} D\hat{y} \qquad \vec{F} = \frac{kq\lambda}{d^2} D\hat{y}$$

if $q, \lambda$ have opposite signs, $\vec{F}$ acts as a linear restoring force like $\vec{F} = -(\text{spring constant})(\text{displacement})$

Hooke's Law