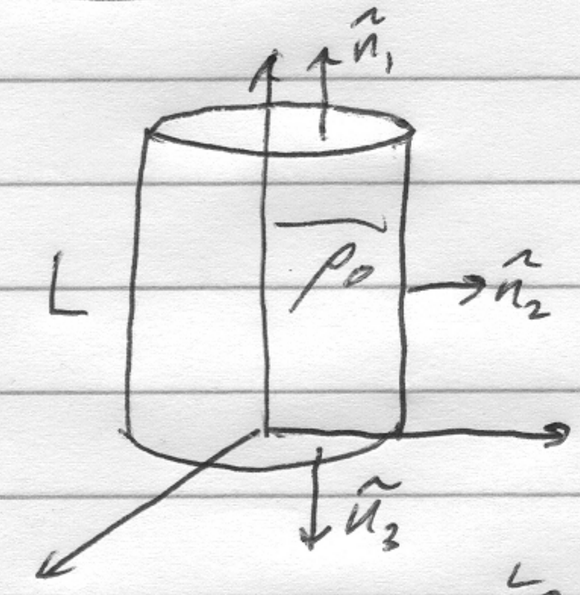
PHYS 3305 HW #2 Solutions, Fall 2013

W 1-22 $\vec{A} = a\rho\hat{\rho} + b\hat{\varphi} + cz\hat{z}$



$\hat{n}_1 = \hat{z}$ $\hat{n}_2 = \hat{\rho}$ $\hat{n}_3 = -\hat{z}$

$$\oint_S \vec{A}\cdot d\vec{a} = \int_{S_1} \vec{A}\cdot\hat{n}_1\,da + \int_{S_2} \vec{A}\cdot\hat{n}_2\,da + \int_{S_3} \vec{A}\cdot\hat{n}_3\,da$$

(1) (2) (3)

$$(1) = \int_0^{2\pi}\int_0^{\rho_0} cz\,\rho\,d\rho\,d\varphi \;(z = L) = cL\pi\rho_0^2$$

$$(2) = \int_0^{L}\int_0^{2\pi} a\rho\,\rho_0\,d\varphi\,dz \;(\rho = \rho_0) = 2\pi a\rho_0^2 L$$

$$(3) = -\int\int cz\,\rho\,d\rho\,d\varphi \;(z = 0) = 0$$

So

$$\oint_S \vec{A}\cdot d\vec{a} = \pi\rho_0^2 L(c + 2a)$$

$$\nabla\cdot\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$= \frac{a}{\rho}\frac{\partial}{\partial\rho}(\rho^2) + 0 + c = 2a + c$$

Constant, so

$$\int_V (\nabla\cdot\vec{A})\,d\tau = (2a+c)(\text{volume})$$

$$= \pi\rho_0^2 L(c + 2a)$$