5. Wang 1-12

$\vec{r} = r\hat{r}$

$= a\hat{r}$

on surface

Surface area element

$d\vec{a} = a^2 \sin\theta \, d\theta \, d\varphi \, \hat{r}$

So $\oint_S \vec{r}\cdot d\vec{a} = a \oint_S da = a^3 \oint_S \sin\theta \, d\theta \, d\varphi$

$\oint_S \vec{r}\cdot d\vec{a} = \underline{4\pi a^3}$

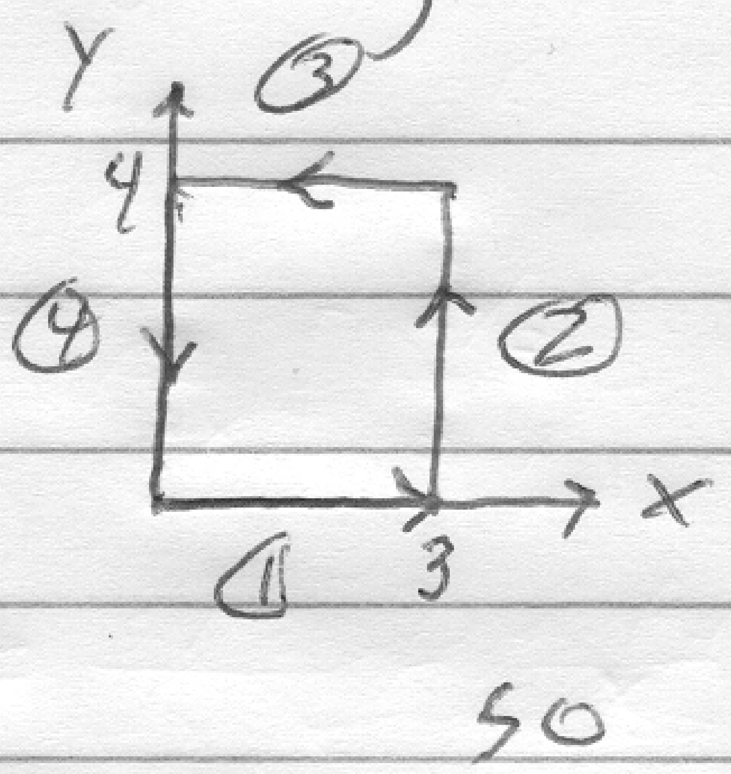
$\nabla\cdot\vec{r} = 3$ since $\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\cdot\left(x\hat{x} + y\hat{y} + z\hat{z}\right)$

$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$

So $\int_V (\nabla\cdot\vec{r})\, d\tau$

$= 3\int_V d\tau = 3\left(\frac{4\pi a^3}{3}\right) = \underline{4\pi a^3}$

6. Wang 1-14 $\vec{A} = -y\hat{x} + x\hat{y}$



$\oint_C \vec{A}\cdot d\vec{s} = \int_1 \vec{A}\cdot d\vec{s} + \int_2 \vec{A}\cdot d\vec{s} + \int_3 \vec{A}\cdot d\vec{s} + \int_4 \vec{A}\cdot d\vec{s}$

$\underline{1}$ $d\vec{s} = dx\,\hat{x}$ $\vec{A}\cdot d\vec{s} = -y\,dx$ but $y=0$

so $\int_1 \vec{A}\cdot d\vec{s} = 0$

2 $d\vec{s} = dy\,\hat{y}$ $\vec{A}\cdot d\vec{s} = x\,dy = 3\,dy$ so

$\int_2 \vec{A}\cdot d\vec{s} = 3\int_0^4 dy = \underline{12}$

3 $d\vec{s} = -dx\,\hat{x}$ $\vec{A}\cdot d\vec{s} = y\,dx$ $\int_3 \vec{A}\cdot d\vec{s} = \int_0^3 4\,dx = \underline{12}$

4 $d\vec{s} = -dy\,\hat{y}$ $\vec{A}\cdot d\vec{s} = -x\,dy$ but $x=0$ so $\int_4 \vec{A}\cdot d\vec{s} = 0$

and

$\oint_C \vec{A}\cdot d\vec{s} = \underline{24}$