Phys 3305 HW #1 Solutions, Fall 2013

1. If $\vec{A}\cdot\vec{B}=0$, they are perpendicular.

$$\vec{A}\cdot\vec{B}=(\hat{x}+4\hat{y}+3\hat{z})\cdot(4\hat{x}+2\hat{y}-4\hat{z})$$
$$=4+8-12=\underline{0} \quad \text{since } \hat{x}\cdot\hat{y} \text{ etc.}=0.$$

2. $\hat{n}=\frac{\nabla u}{|\nabla u|}$ for $u=ax^2+by^2+cz^2$

$$\nabla u=\left(\frac{\partial}{\partial x}\hat{x}+\frac{\partial}{\partial y}\hat{y}+\frac{\partial}{\partial z}\hat{z}\right)u$$

$$\nabla u=2ax\hat{x}+2by\hat{y}+2cz\hat{z}$$
$$|\nabla u|=\sqrt{(2ax)^2+(2by)^2+(2cz)^2}=2\sqrt{a^2x^2+b^2y^2+c^2z^2}$$

So
$$\underline{\hat{n}=\frac{\nabla u}{|\nabla u|}=\frac{ax\hat{x}+by\hat{y}+cz\hat{z}}{\sqrt{a^2x^2+b^2y^2+c^2z^2}}}$$

3. W 1-6

Verify 1-29

$$\vec{A}\cdot(\vec{B}\times\vec{C})=\vec{A}\cdot\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\B_x&B_y&B_z\\C_x&C_y&C_z\end{vmatrix}=\begin{vmatrix}A_x&A_y&A_z\\B_x&B_y&B_z\\C_x&C_y&C_z\end{vmatrix}$$

Switch 1st + 3rd rows

$$=-\begin{vmatrix}C_x&C_y&C_z\\B_x&B_y&B_z\\A_x&A_y&A_z\end{vmatrix}$$ Now switch 2nd + 3rd

$$=+\begin{vmatrix}C_x&C_y&C_z\\A_x&A_y&A_z\\B_x&B_y&B_z\end{vmatrix}=\vec{C}\cdot(\vec{A}\times\vec{B})=\underline{(\vec{A}\times\vec{B})\cdot\vec{C}}$$