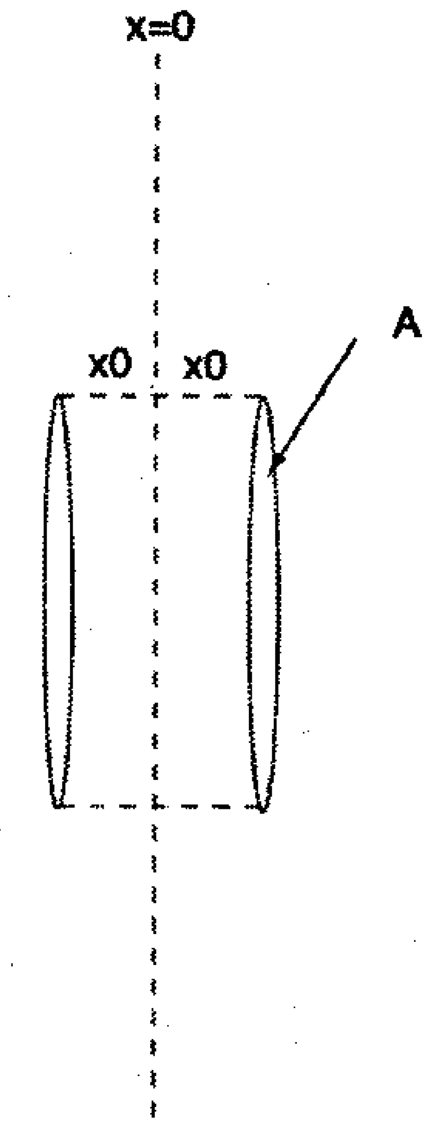
2. A planar symmetry volume of charge is centered on the y-z plane; that is, its plane of symmetry is the x=0 plane. It has a volume charge density given by $\rho(x) = \beta \exp(-\frac{|x|}{a})$, where β and a are constants.




a) (10 points) Find the charge per unit area contained within a region between $+x_0$ and $-x_0$. (Hint: Construct a cylindrical pillbox Gaussian surface with its flat sides of area A parallel to the y-z plane, centered on x=0, extending a distance $x_0$ on either side of x=0).

$$dV = A\,dx$$

$$q_{in} = \int \rho\, dV = 2\beta A \int^{x_0} e^{-x/a} dx$$

$$q_{in} = -2\beta a A\, e^{-x/a}\Big|_0^{x_0} = 2\beta a A\left(1-e^{-x_0/a}\right)$$

$$\frac{q_{in}}{A} = 2\beta a\left(1-e^{-x/a}\right) \qquad (x_0 \to x)$$

b) (5 points) Use Gauss' Law to find the electric field as a function of x.

$$2EA = q_{in}/\varepsilon_0 = \frac{1}{\varepsilon_0} 2\beta a A\left(1-e^{-x/a}\right)$$

$$E = \frac{\beta a}{\varepsilon_0}\left(1-e^{-x/a}\right)$$

c) (5 points) What is the value of E when x goes to infinity? Does this make sense in view of the infinitely thin sheet result presented in class?

$x \to \infty \quad E \to \frac{\beta a}{\varepsilon_0}, \quad \sigma = \frac{q_{in}}{A} \to 2\beta a$ as $x \to \infty$

So $E \to \frac{\sigma}{2\varepsilon_0}$ as for thin sheet.

3. The potential of a ring of charge with radius a and charge Q on-axis a distance x from the center is given by $V(x) = \frac{k_e Q}{\sqrt{x^2+a^2}}$. Suppose a = 0.30 m and Q = 45 microCoulombs.

$m = 0.0060$ kg

a) If a +2.0 microCoulomb charge, placed at the center of the ring is released (with a teeny nudge), what will its speed be when it has reached x=2a, if it is constrained to move on the x axis?

$$\Delta V = V_f - V_i = k_e Q\left[\frac{1}{\sqrt{(2a)^2+a^2}} - \frac{1}{a}\right] = k_e Q\left[\frac{1}{\sqrt{5}} - 1\right]\frac{1}{a}$$

$$\Delta K = -q\Delta V = \frac{k_e q Q}{a}\left[1 - \frac{1}{\sqrt{5}}\right] = 1.49\text{ J} = \frac{1}{2} m v_f^2$$

$$v_f = \sqrt{\frac{2(1.49\text{ J})}{6\times10^{-3}\text{ kg}}} = 22.3\text{ m/s}$$

b) Use the relationship between potential and electric field to show that a negative charge at the center will feel an approximately linear restoring force (pulling it back to the center) if it is displaced by a distance x along the axis where x<<a.

$$E_x = -\frac{\partial V}{\partial x} = -k_e Q\left(-\frac{1}{2}\right)\frac{2x}{(x^2+a^2)^{3/2}} = \frac{k_e Q x}{(x^2+a^2)^{3/2}}$$

q negative

$$F_x = -\frac{k_e Q |q|}{(x^2+a^2)^{3/2}}\, x$$