Physics 1403-002 Exam #3, Spring 2008

Instructions: Show your work for all problems. Partial credit will be assigned for things that make sense. 

\[ \text{g} = 9.81 \text{ m/s}^2, \quad \rho_{\text{water}} = 1000 \text{ kg/m}^3, \quad \rho_{\text{air}} = 1.29 \text{ kg/m}^3, \quad P_{\text{atm}} = 1.013 \times 10^5 \text{ Pa}. \]

All questions are equally weighted (so they are 8.333 points each). For numerical answers, PLEASE write your answer on the right of the problem with a box around it. Your low question will be dropped.

1. A cylinder and a hoop which have the same mass and radius are rolling on a horizontal surface without slipping. They have the same kinetic energy. Which rolls faster, and why?

\[ K = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 = \frac{1}{2} (Ie + mR^2) \omega^2 \text{ since } \omega = \frac{v}{R} \]

If \( K \) is same, the one with the smaller \( I/e \) will go faster.

**Cylinder**

2. A rotating sphere with \( M = 2.5 \text{ kg} \) and radius \( R = 0.33 \text{ m} \) goes from an angular velocity of 2.0 \( \text{ rad/s} \) to an angular velocity of 5.0 \( \text{ rad/s} \) in a time period of 1.2 seconds. What is the magnitude of the torque acting on the sphere?

\[ \tau_{\text{avg}} = \frac{I \Delta \omega}{\Delta t} = \frac{5 \times 10 \times 2 - 2 \times 1.6}{1.2} = 2.5 \text{ Nm} \]

So \( \tau = I \omega \)

\[ I = \frac{2}{5} mR^2 = 0.109 \text{ kg} \cdot \text{m}^2 \]

So \( \tau = I \omega = (0.109 \text{ kg} \cdot \text{m}^2)(2.5 \text{ rad/s}) = 0.27 \text{ Nm} \)

3. A freely rotating horizontal disc with a moment of inertia of \( I = 3.5 \times 10^3 \text{ kg} \cdot \text{m}^2 \) is initially rotating at an angular frequency of 8.0 \( \text{ rad/s} \). A hoop with a radius of 0.25 \( \text{ m} \) and a mass of 0.20 \( \text{ kg} \) is held horizontally above the disc, then dropped onto the disc so that the centers of the disc and the hoop coincide. They collide and stick. What is the final angular velocity of the system?

Conservation of angular momentum:

\[ I_1 = \frac{3}{2} \text{ kg} \cdot \text{m}^2, \quad I_2 = mR^2 = 1.25 \times 10^{-2} \text{ kg} \cdot \text{m}^2 \]

\[ L_i = I_1 \omega_i \]

\[ L_f = (I_1 + I_2) \omega_f \]

So \( I \omega_f = \frac{I}{1 + I_2} \omega_i = 1.75 \text{ rad/s} \)

4. In the diagram on the right, the horizontal surface is frictionless and gravity acts downwards. \( M_1 = 6.5 \text{ kg} \) and \( M_2 = 2.8 \text{ kg} \) and are connected with a string, while the pulley that the string goes over has a radius of 0.35 \( \text{ m} \) and a moment of inertia of 0.150 \( \text{ kg} \cdot \text{m}^2 \). If the masses are released from rest, find the speed of the masses after \( M_2 \) has dropped 1.5 \( \text{ m} \).

\[ \Delta K = -m_2 g h, \quad \Delta K = \frac{1}{2} k \left( v_2 - v_1 \right)^2 + \frac{1}{2} I \omega \]

\( \Delta K = -\Delta U \)

so \( mgh = \frac{1}{2} k v_1^2 + \frac{1}{2} k v_2^2 + \frac{1}{2} I \omega \)

and \( v^2 = \frac{2ghm_2}{k + \frac{1}{2} k \left( \frac{v_1}{v_2} \right)^2} = 7.82 \text{ m/s} \)

\[ v = 2.80 \text{ m/s} \]

5. When an adult and a child sit on opposite sides of a teeter-totter, the adult must sit closer to the pivot than the child in order to make it balance. Explain why this is so.

Since torque \( \tau = Fr \) when \( \theta = 90^\circ \), and the torques must balance, the object with larger weight must be closer to the pivot.

6. A ladder with a mass of 12 kg and a length of 2.5 m leans against a frictionless wall at an angle of 60 degrees above the horizontal. The bottom of the ladder is held in place with an unknown frictional force. Find the force exerted on the ladder by the wall.

\[ N \sin 60^\circ - L = \frac{1}{2} m g \sin 30^\circ = 0 \]

\[ N = \frac{1}{2} \cos 60^\circ \sin 60^\circ = 24 \text{ N} \]