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## UPSILON SPECTROSCOPY FROM LATTICE QCD

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We have calculated the spectrum and wavefunctions of mesons and baryons composed of  $b$  quarks using nonrelativistic QCD on the lattice. We include spin effects to lowest order. Spin-independent quantities and  $s$  state hyperfine splittings are calculated to high accuracy.

### 1. NRQCD

Nonrelativistic QCD is an effective field theory that can be used in lattice simulations of heavy-quark bound states [1]. Comparison with experimental data on, for example, the  $\Upsilon$  family, will, we believe, provide a precise non-perturbative test of QCD. Systematic errors are more readily controlled here than in light-quark calculations and algorithms are fast.

We [2] have used the non-relativistic action for heavy quarks

$$S_Q = \sum_{\mathbf{x},t} \psi_{\mathbf{x},t}^\dagger \left\{ \Delta_4 + H - z \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2Ma} \right\} \psi_{\mathbf{x},t}, \quad (1)$$

$$H = - \sum_{j=1}^3 \frac{\Delta_j \Delta_{-j}}{2Ma}. \quad (2)$$

$M$  is the heavy quark mass and  $a$ , the lattice spacing.  $z$  is the coupling of the  $\boldsymbol{\sigma} \cdot \mathbf{B}$  term; it can be calculated in perturbation theory by matching to full QCD.  $\Delta_\mu$  and  $\Delta_{-\mu}$  are the standard forward and backward covariant finite difference operators.  $\psi$  is a 2-component spinor field representing the quark. The zero of energy has been

shifted to remove the usual mass term.

Using this action we can solve for the quark propagator in a background gauge field.

$$G_{\mathbf{x},t+1} = U_{\mathbf{x},t,4}^\dagger \left[ \left( 1 - \frac{H}{2} \right)^2 + z \frac{\boldsymbol{\sigma} \cdot \mathbf{B}}{2Ma} \right] G_{\mathbf{x},t}, \quad (3)$$

$$G_{\mathbf{x},t} = 0, \quad t \leq 0, \quad (4)$$

$$G_{\mathbf{x},t} = U_{\mathbf{0},0}^\dagger \delta_{\mathbf{x},0}, \quad t = 1. \quad (5)$$

The reason that the  $H$  term is 'halved and squared' is to avoid a numerical instability that occurs when  $Ma < 3$  if this is not done [1]. Note that the quark propagator can be calculated on one sweep through the lattice.

For background gauge fields we used  $30 \cdot 16^3 \times 24$  quenched configurations generously provided by the Staggered Collaboration. We fixed these to lattice Coulomb gauge using a Fourier accelerated steepest descents algorithm. The lattice operator that we chose to represent the chromomagnetic field,  $B_j(\mathbf{x})$ , is the standard traceless sum of the four plaquettes bordering the point  $\mathbf{x}$  in the plane perpendicular to  $j$ .

To measure meson and baryon correlation functions we combined quark and antiquark

propagators using appropriate vertex operators to couple to the different states. The simplest local vertex operators given in Reference [1] were used. We calculated correlation functions for the following mesons:  $^1S_0$ ,  $^3S_1$ ,  $^1P_1$ ,  $^3P_0$ ,  $^3P_1$  and  $^3P_2$  and also for two baryons composed of  $b$  quarks — the spin  $\frac{3}{2}$  and the fictitious spin  $\frac{1}{2}$ . For meson and baryon wavefunctions we used the local operators at the source end and operators which extend over a number of lattice spacings at the sink end.

We fitted the average correlation function to  $Ae^{-mt}$  at large times,  $t$ .  $m$  is the lightest mass in that channel, i.e. the one with radial quantum number  $n = 1$ . For mass splittings the same form was fitted to the ratio of average correlation functions.  $\chi^2$  was minimised using the covariance matrix inverted with singular value decomposition to remove spurious eigenvectors. Errors on the effective mass were obtained using the bootstrap method.

Both the heavy quark mass,  $M$ , and the lattice spacing,  $a$ , must be fixed by reference to experiment. We have used two different values for  $Ma$ , 1.7 and 2.3, since we believe this should cover the range for the  $b$  quark at  $\beta = 6.0$ . We also take two values for  $z$ , 1.0 and 1.35, to see how the quantities measured depend on  $z$ .

## 2. Numerical results

The  $s$  state meson correlation functions are determined very accurately. Unfortunately the effective mass converges only very slowly to a plateau and exhibits oscillations. These effects make it hard to fit the correlation function to the desired form. If instead we fit the hyperfine splitting  $\Delta m(^3S_1 - ^1S_0)$  directly both problems disappear and a plateau in the effective mass is seen over 40 time slices (see Figure 1).

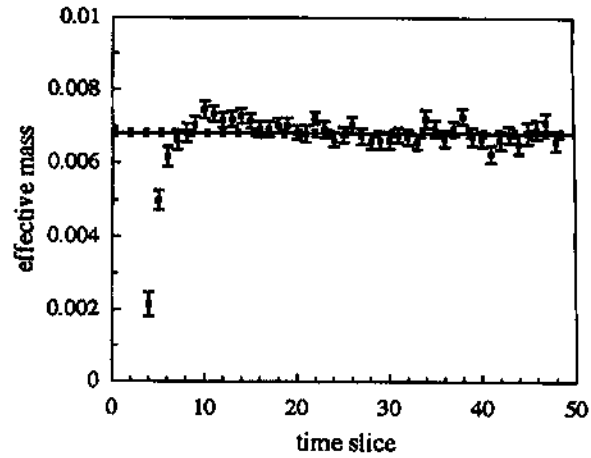


Fig. 1. The effective mass splitting between the  $\Upsilon$  and the  $\eta_b$  as a function of time slice on the lattice.  $\beta = 6.0$ ,  $Ma = 2.3$ ,  $z = 1.0$ . The solid line represents the fitted mass and the dashed lines give the range for  $\Delta\chi^2 = 1$ .

The  $p$  and  $d$  state meson correlation functions are much noisier than the  $s$  state, since the signal/noise ratio falls exponentially with the mass splitting from the  $s$  state. All the  $p$  states are degenerate within errors.

The spin-independent quantity of most interest [3] is the spin-averaged  $s$ - $p$  splitting. Each state is weighted in the average by the number of spin components. Our results give a value which is the same for each data set within errors, indicating that the result is independent of  $Ma$ , in agreement with experiment, and  $z$ . We obtain  $0.23 \pm 0.01$  to be compared to an experimental value of  $452 \pm 4$  MeV. We can thus obtain an accurate value for the lattice spacing and thence the scale of QCD, since the splitting depends on no other parameters. The result,  $a^{-1} = 2.0 \pm 0.1$  GeV, can be corrected [3] to  $a^{-1} = 2.1 \pm 0.2$  GeV by allowing for systematic errors which will be removed in future calculations.

Using this value for the lattice spacing we can now convert mass splittings into physical units. Our data show that the  $s$  state hyperfine splitting,  $m(\Upsilon) - m(\eta_b)$ , is accurately proportional to  $z^2$  and approximately inversely proportional to  $M$ . We obtain values ranging from 14 to 26 MeV. Given values for  $Ma$  and  $z$  we could give an accurate prediction for the  $\eta_b$  mass. The baryonic hyperfine splitting is roughly  $\frac{2}{5}$  that of the mesonic one.

We have also calculated the splitting between the  $^1D_2$  and the  $^1S_0$  states. Using the experimental  $\Upsilon$  mass and the lattice spacing above, we predict a  $^1D_2$  mass of 10.4 GeV. The  $^3D_1$  meson, which couples directly to  $e^+e^-$ , should lie close by.

To derive physical masses for the heavy baryons we need to fix the zero of energy. We find an energy shift of 3.6 GeV per quark is required to match our data to experiment for the  $\Upsilon$  mass. This shift is considerably smaller than that calculated in perturbation theory. Applying three times this shift to the measured baryon masses gives a physical mass for the  $\frac{3}{2}$  baryon of  $14.31 \pm 0.01$  GeV.

We have measured wavefunctions for the  $^3S_1$  and  $^1P_1$  mesons and the  $\frac{3}{2}$  baryon. The radial wavefunctions for the  $^3S_1$  and the  $^1P_1$  are shown in Figure 2. They resemble, at least qualitatively, those obtained from potential models.

### 3. Conclusions

Calculations using NRQCD on the lattice look very promising as a way of testing QCD. Future calculations must extend the action to study further splittings and use smeared operators to improve the measurements and look for excited states.

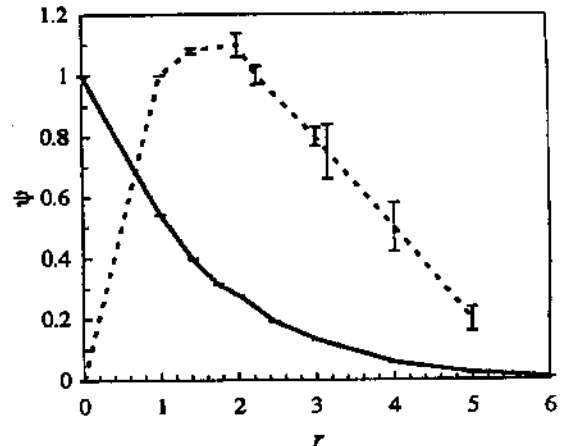


Fig. 2. Radial wavefunctions for the  $^3S_1$  (solid line) and  $^1P_1$  (dashed line) mesons plotted against separation of the quark and antiquark in lattice units.  $\beta = 6.0$ ,  $Ma = 2.3$  and  $z = 1.0$ . The  $^3S_1$  wavefunction is normalised to take the value 1.0 at the origin. The  $^1P_1$  wavefunction is normalised to take the value 1.0 at  $r = 1$  lattice spacing.

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### References

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