Capacitance of a Parallel Plate Capacitor

The capacitance of a capacitor is defined as

$$C = \frac{Q}{\Delta V}$$

For a parallel plate capacitor, the potential difference (the work needed to move a charge from one plate to another) between the plates is given by

$$\Delta V = -\frac{W}{q}$$

$$\Delta V = -\frac{Fd}{q} = -\frac{qEd}{q}$$

$$\Delta V = -Ed$$

where W is the work to move a charge from one plate to another, F = qE is the force of the electric field between the plates, and d is the distance between plates.

The potential difference is proportional to the distance, d, between the plates. Since the amount of charge on the plates, Q, depends on the area, A, of the plates, and it is reasonable to believe the capacitance on the area of the plates and the distance between the plates.

It can be shown mathematically and experimentally that capacitance is proportional to the ratio of the area of the plates to the distance between the plates, $\frac{A}{d}$. The capacitance of a parallel plate capacitor is

$$C = \frac{\varepsilon_0 A}{d}$$

where A is the area of the plates, d is the distance between the plates, and ε_0 is a constant. The value of ε_0 is $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / Nm^2$.

Potential Energy of a Charged Capacitor

In the process of charging a capacitor, work is done to move the charges on the plates of the capacitor. The total work required to charge a capacitor can be calculated. It is found to be

$$W = \frac{Q^2}{2C}$$

Since work is equal to a change in potential energy for conservative forces, the electrical potential energy of a capacitor increases by $\frac{Q^2}{2C}$ when it is charged. This potential energy would decrease and work would be done, if the capacitor were discharged.