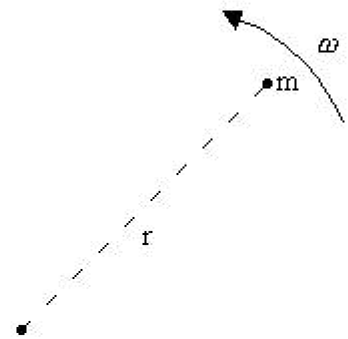
UNIT 16 READING A

Moment of inertia and rotational kinetic energy

Consider an object of mass m moving in a circle about an axis as in the picture below.



The kinetic energy of the object is

$$K = \frac{1}{2}mv^2$$

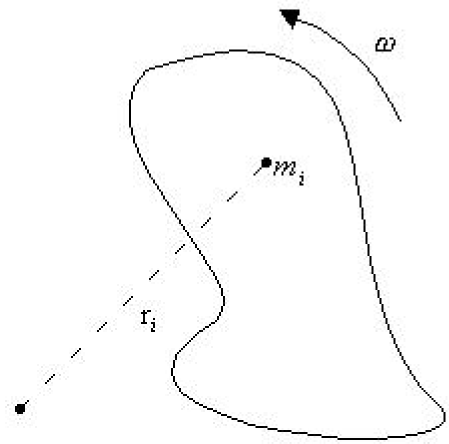
This can be written in terms of the angular speed of the object, ω, and the distance of the object from the axis of rotation, r

$$K = \frac{1}{2}mv^2$$
$$K = \frac{1}{2}m\left(r\omega\right)^2$$
$$K = \frac{1}{2}\left(mr^2\right)\omega^2$$

The quantity mr^2 is defined to be the moment of inertia of the mass m. The kinetic energy can then be rewritten in terms on the moment of inertia

$$K = \frac{1}{2}I\omega^2$$

For a large object, we can consider it made up of many smaller pieces of mass.



Each piece of mass, m_i, is at a different distance, r_i, from the axis of rotation. The kinetic energy of the object is found by summing up the kinetic energies of each the small pieces of mass:

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

This can be written as

$$K = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$$

$$K = \frac{1}{2} I \omega^2$$

where $I = \sum_i m_i r_i^2$ is the moment of inertia of the object. This is the rotational kinetic energy of an object.

A rolling object has both rotational and translational kinetic energy, the total kinetic energy for a rolling object is

$$K_{total} = \frac{1}{2} I \omega^2 + \frac{1}{2} M v_{CM}^2$$

where I is the moment of inertia of the object, ω is the angular speed of the object, M is the mass of the object and v_{CM} is the velocity of the center of mass of the object.