UNIT 13 READING A

(Raymond A. Survey & Robert J. Beichner, Physics for Science and Engineers, Saunders College Publishing, Fort Worth, TX, 2000.)

17.5 THE DOPPLER EFFECT

Perhaps you have noticed how the sound of a vehicle's horn changes as the vehicle moves past you. The frequency of the sound you hear as the vehicle approaches you is higher than the frequency you hear as it moves away from you (see Quick-Lab). This is one example of the **Doppler effect.**[3]

To see what causes this apparent frequency change, imagine you are in a boat that is lying at anchor on a gentle sea where the waves have a period of $T = 3.0$ s. This means that every 3.0 s a crest hits your boat. Figure 17.9a shows this situation, with the water waves moving toward the left. If you set your watch to $t = 0$ just as one crest hits, the watch reads 3.0 s when the next crest hits, 6.0 s when the third crest hits, and so on. From these observations you conclude that the wave frequency is $f = 1/T = (1/3.0)$ Hz. Now suppose you start your motor and head directly into the oncoming waves, as shown in Figure 17.9b. Again you set your watch to $t = 0$ as a crest hits the front of your boat. Now, however, because you are moving toward the next wave crest as it moves toward you, it hits you less than 3.0 s after the first hit. In other words, the period you observe is shorter than the 3.0-s period you observed when you were stationary. Because $f = 1/T$, you observe a higher wave frequency than when you were at rest.

If you turn around and move in the same direction as the waves (see Fig. 17.9c), you observe the opposite effect. You set your watch to $t = 0$ as a crest hits the back of the boat. Because you are now moving away from the next crest, more than 3.0 s has elapsed on your watch by the time that crest catches you. Thus, you observe a lower frequency than when you were at rest.

These effects occur because the relative speed between your boat and the waves depends on the direction of travel and on the speed of your boat. When you are moving toward the right in Figure 17.9b, this relative speed is higher than that of the wave speed, which leads to the observation of an increased frequency. When you turn around and move to the left, the relative speed is lower, as is the observed frequency of the water waves.

Let us now examine an analogous situation with sound waves, in which the water waves become sound waves, the water becomes the air, and the person on the boat becomes an observer listening to the sound. In this case, an observer O is moving and a sound source S is stationary. For simplicity, we assume that the air is also stationary and that the observer moves directly toward the source. The observer moves with a speed v_O toward a stationary point source ($v_S = 0$) (Fig. 17.10). In general, *at rest* means at rest with respect to the medium, air.

[3] Named after the Austrian physicist Christian Johann Doppler (1803–1853), who discovered the effect for light waves.

(a)

(b)

(c)

Figure 17.9 (a) Waves moving toward a stationary boat. The waves travel to the left, and their source is far to the right of the boat, out of the frame of the drawing. (b) The boat moving toward the wave source. (c) The boat moving away from the wave source.

We take the frequency of the source to be f, the wavelength to be λ, and the speed of sound to be v. If the observer were also stationary, he or she would detect f wave fronts per second. (That is, when $v_O = 0$ and $v_S = 0$, the observed frequency equals the source frequency.) When the observer moves toward the source,

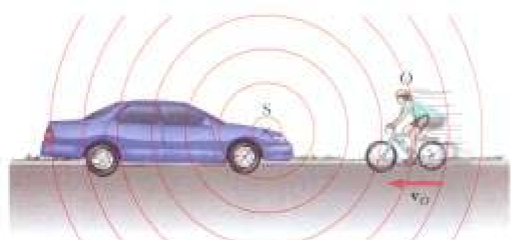


Figure 17.10 An observer O (the cyclist) moves with a speed v_O toward a stationary point source S, the horn of a parked car. The observer hears a frequency f' that is greater than the source frequency.

the speed of the waves relative to the observer is $v' = v + v_O$, as in the case of the boat, but the wavelength λ is unchanged. Hence, using Equation 16.14, $v = \lambda f$, we can say that the frequency heard by the observer is *increased* and is given by

$$f' = \frac{v'}{\lambda} = \frac{v + v_O}{\lambda}$$

Because $\lambda = v/f$, we can express f' as

$$f' = \left(1 + \frac{v_O}{v}\right)f \qquad \text{(observer moving toward source)} \qquad \textbf{(17.11)}$$

If the observer is moving away from the source, the speed of the wave relative to the observer is $v' = v - v_O$. The frequency heard by the observer in this case is *decreased* and is given by

$$f' = \left(1 - \frac{v_O}{v}\right)f \qquad \text{(observer moving away from source)} \qquad \textbf{(17.12)}$$

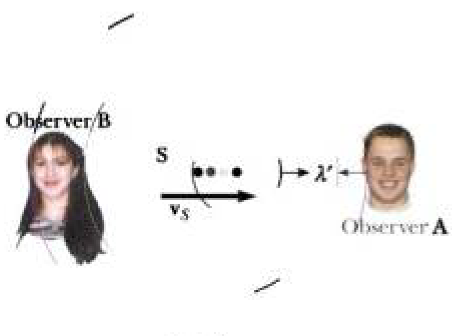
In general, whenever an observer moves with a speed v_O relative to a stationary source, the frequency heard by the observer is

Frequency heard with an observer in motion

$$f' = \left(1 \pm \frac{v_O}{v}\right)f \qquad \textbf{(17.13)}$$

where the positive sign is used when the observer moves toward the source and the negative sign is used when the observer moves away from the source.

Now consider the situation in which the source is in motion and the observer is at rest. If the source moves directly toward observer A in Figure 17.11a, the wave fronts heard by the observer are closer together than they would be if the source were not moving. As a result, the wavelength λ' measured by observer A is shorter than the wavelength λ of the source. During each vibration, which lasts for a time T (the period), the source moves a distance $v_S T = v_S/f$ and the wavelength is



(a)

(b)

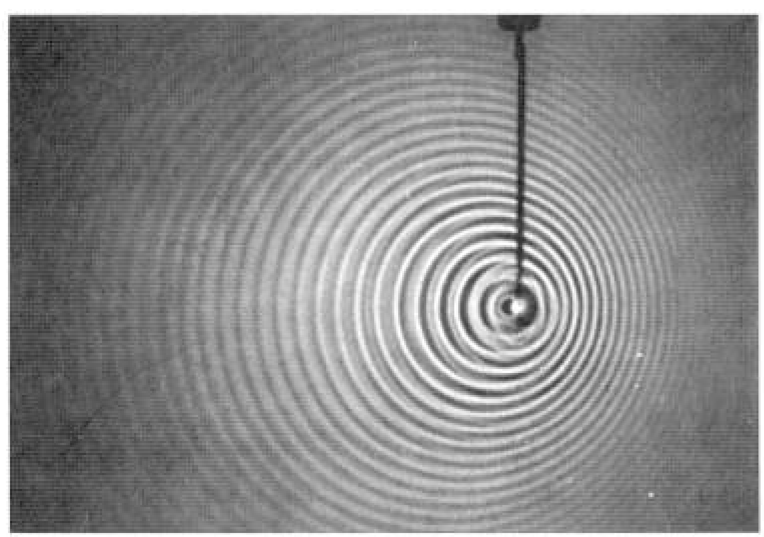


Figure 17.11 (a) A source S moving with a speed v_S toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water, observed in a ripple tank. A point source is moving to the right with speed v_S. *(Courtesy of the Educational Development Center, Newton, MA)*

shortened by this amount. Therefore, the observed wavelength λ' is

$$\lambda' = \lambda - \Delta\lambda = \lambda - \frac{v_S}{f}$$

Because $\lambda = v/f$, the frequency heard by observer A is

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - \dfrac{v_S}{f}} = \frac{v}{\dfrac{v}{f} - \dfrac{v_S}{f}}$$

$$f' = \left(\frac{1}{1 - \dfrac{v_S}{v}}\right) f \qquad (17.14)$$

"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect." *(Sydney Harris)*

That is, the observed frequency is *increased* whenever the source is moving toward the observer.

When the source moves away from a stationary observer, as is the case for observer B in Figure 17.11a, the observer measures a wavelength λ' that is *greater* than λ and hears a *decreased* frequency:

$$f' = \left(\frac{1}{1 + \dfrac{v_S}{v}}\right) f \qquad (17.15)$$

Combining Equations 17.14 and 17.15, we can express the general relationship for the observed frequency when a source is moving and an observer is at rest as

Frequency heard with source in motion

$$f' = \left(\frac{1}{1 \mp \dfrac{v_S}{v}}\right) f \qquad (17.16)$$

Finally, if both source and observer are in motion, we find the following general relationship for the observed frequency:

Frequency heard with observer and source in motion

$$f' = \left(\frac{v \pm v_O}{v \mp v_S}\right) f \qquad (17.17)$$

In this expression, the upper signs ($+v_O$ and $-v_S$) refer to motion of one toward the other, and the lower signs ($-v_O$ and $+v_S$) refer to motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler-effect problems is as follows:

The word *toward* is associated with an *increase* in observed frequency. The words *away from* are associated with a *decrease* in observed frequency.

Although the Doppler effect is most typically experienced with sound waves, it is a phenomenon that is common to all waves. For example, the relative motion of source and observer produces a frequency shift in light waves. The Doppler effect is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the speeds of stars, galaxies, and other celestial objects relative to the Earth.