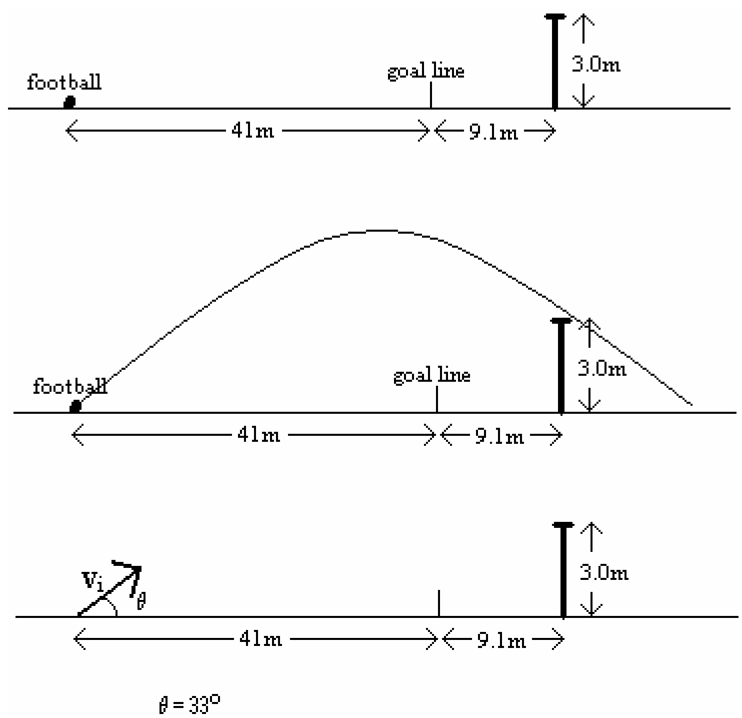
UNIT 3 READING B

Example of a two-dimensional motion problem using the problem solving strategy:

The Texas A&M – Texas Tech football game is tied and only enough time remains on the clock for one last field goal attempt by the Red Raiders. The ball will be kicked from the 45-yard (41m) line. The cross bar of the goal post is 10ft (3.0m) high and lies 10 yards (9.1m) behind the goal line. If the ball leaves the kicker's foot at an angle of 33^o to the ground, what initial velocity is required in order for Tech to win the ball game? Show your work.

(1) **Focus on the Problem:** What's going on?

The football player is going to kick the football. The football will follow a curved path as in the middle picture below. The bottom picture below illustrates the velocity of the football after it is kicked and the angle at which it leaves the ground.



We need to find the velocity of the football as it leaves the ground. Since this is a two-dimensional motion problem, we will need to consider both the x- and y- components of the motion. We will need to use the following equations:

$$x_f - x_i = \frac{v_{xf} + v_{xi}}{2} t \qquad y_f - y_i = \frac{v_{yf} + v_{yi}}{2} t$$

$$v_{xf} - v_{xi} = \bar{a}_x t \qquad v_{yf} - v_{yi} = \bar{a}_y t$$

$$x_f - x_i = v_{xi} t + \frac{1}{2}\bar{a}_x t^2 \qquad y_f - y_i = v_{yi} t + \frac{1}{2}\bar{a}_y t^2$$

$$2\bar{a}_x (x_f - x_i) = v_{xf}^2 - v_{xi}^2 \qquad 2\bar{a}_y (y_f - y_i) = v_{yf}^2 - v_{yi}^2$$

In order for the football to pass over the goalpost, the y-component of the position of the football must be 3.0m at the same time that the x-component of the position of the football is 50.1m.

(2) **Describe the Physics:** What is the physics?

The football has an initial velocity both in the x- and y- directions. We can express the components of the initial velocity in terms of the velocity, v_i, and the angle, θ. As the football is in flight, there is no acceleration in the x-direction. The acceleration in the y-direction is -9.8m/s^2. The y-component of the position of the football is 3.0m at the same time that the x-component of the position of the football is 50.1m. The initial position of the football is at x = 0m and y = 0m. In symbols:

$$x_i = 0m \qquad y_i = 0m$$

$$x_f = 50.1m \qquad y_f = 3.0m$$

$$\bar{a}_x = 0 \qquad \bar{a}_y = -9.8 m/s^2$$

$$v_{xi} = v_i \cos\theta \qquad v_{yi} = v_i \sin\theta$$

$$v_{xf} = ? \qquad v_{yf} = ?$$

$$t = ? \qquad t = ?$$

We need to use the above quantities and the equations mentioned earlier to find v_i.

(3) **Plan the Solution:** Can we find the solution?

Using the fact that $\bar{a}_x = 0$, $v_{xi} = v_i \cos\theta$, and $v_{yi} = v_i \sin\theta$, the equations in the x- and y- directions become:

$$v_{xf} = v_{xi} = v_i \cos\theta$$

$$x_f - x_i = (v_i \cos\theta) t$$

$$y_f - y_i = \frac{v_{yf} + v_i \sin\theta}{2} t$$

$$v_{yf} - v_i \sin\theta = \bar{a}_y t$$

$$y_f - y_i = (v_i \sin\theta) t + \frac{1}{2}\bar{a}_y t^2$$

$$2\bar{a}_y (y_f - y_i) = v_{yf}^2 - (v_i \sin\theta)^2$$

Since v_{yf} is not known and is not a quantity we need to solve the problem, we focus on the following two equations, which contain the initial velocity, v_i, which we are looking for and the time:

$$x_f - x_i = (v_i \cos\theta)t$$

$$y_f - y_i = (v_i \sin\theta)t + \frac{1}{2}\bar{a}_y t^2$$

We now have two equations and two unknowns and we can solve the problem.

(4) **Execute the Plan:** What is the answer?

Since $y_f = 3.0$m at the same time that the $x_f = 50.1$m, the time is the same in each of these equations. We can solve one of the equations for the time and plug it into the second equation, and solve for v_i:

$$t = \frac{x_f - x_i}{v_i \cos\theta}$$

$$y_f - y_i = (v_i \sin\theta)\left(\frac{x_f - x_i}{v_i \cos\theta}\right) + \frac{1}{2}\bar{a}_y\left(\frac{x_f - x_i}{v_i \cos\theta}\right)^2$$

$$y_f - y_i = \tan\theta(x_f - x_i) + \frac{1}{2}\bar{a}_y\left(\frac{x_f - x_i}{v_i \cos\theta}\right)^2$$

$$\frac{1}{2}\bar{a}_y\left(\frac{x_f - x_i}{v_i \cos\theta}\right)^2 = y_f - y_i - \tan\theta(x_f - x_i)$$

$$\left(\frac{x_f - x_i}{v_i \cos\theta}\right)^2 = 2\left(\frac{y_f - y_i - \tan\theta(x_f - x_i)}{\bar{a}_y}\right)$$

$$\frac{x_f - x_i}{v_i \cos\theta} = \sqrt{2\left(\frac{y_f - y_i - \tan\theta(x_f - x_i)}{\bar{a}_y}\right)}$$

$$\frac{1}{v_i} = \frac{\cos\theta}{x_f - x_i}\sqrt{2\left(\frac{y_f - y_i - \tan\theta(x_f - x_i)}{\bar{a}_y}\right)}$$

$$v_i = \frac{1}{\dfrac{\cos\theta}{x_f - x_i}\sqrt{2\left(\dfrac{y_f - y_i - \tan\theta(x_f - x_i)}{\bar{a}_y}\right)}}$$

$$v_i = \frac{1}{\dfrac{\cos 33^o}{50.1\text{m} - 0\text{m}}\sqrt{2\left(\dfrac{3.0\text{m} - 0\text{m} - \tan 33^o(50.1\text{m} - 0\text{m})}{-9.8\text{m/s}^2}\right)}}$$

$$v_i = 24 m/s$$

The initial velocity of the football must be 24m/s in order for the football to just pass over the goal post.

(5) **Evaluate the Answer:** Can this be true?

The units are correct and this is not an unreasonable answer.