UNIT 3 READING A

Two-Dimensional Motion

The position of an object can be represented by a vector, as described in Unit 1 Reading B. The velocity of an object is then defined by:

$$\mathbf{v} = \frac{\mathbf{x}_f - \mathbf{x}_i}{t_f - t_i}$$

where $\mathbf{x}_f$ is the final position vector, $\mathbf{x}_i$ is the initial position vector, $t_f$ is the final time, and $t_i$ is the initial time. The velocity, $\mathbf{v}$, is the rate of change of the position vector. Because this is a vector equation, it is very hard to use in this form and it is easier to understand, if it is separated into its x- and y- components:

$$\mathbf{v}_x = \frac{x_f - x_i}{t_f - t_i}$$

$$\mathbf{v}_y = \frac{y_f - y_i}{t_f - t_i}$$

The acceleration is defined as

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i}$$

where $\mathbf{v}_f$ is the final velocity vector, $\mathbf{v}_i$ is the initial velocity vector, $t_f$ is the final time, and $t_i$ is the initial time. The acceleration, $\mathbf{a}$, is the rate of change of the velocity vector. Because this is a vector equation, it is very hard to use in this form and it is easier to understand, if it is separated into its x- and y- components:

$$\mathbf{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

$$\mathbf{a}_y = \frac{v_{yf} - v_{yi}}{t_f - t_i}$$

If the acceleration and the velocity are separated into their x- and y- components, then the equations of motion can be applied independently to the x- and y- directions:
\[x_f - x_i = \frac{v_{xf} + v_{xi}}{2} t\]

\[v_{xf} - v_{xi} = \bar{a}_x t\]

\[x_f - x_i = v_{xi} t + \frac{1}{2} \bar{a}_x t^2\]

\[2\bar{a}_x (x_f - x_i) = v_{xf}^2 - v_{xi}^2\]

\[y_f - y_i = \frac{v_{yf} + v_{yi}}{2} t\]

\[v_{yf} - v_{yi} = \bar{a}_y t\]

\[y_f - y_i = v_{yf} t + \frac{1}{2} \bar{a}_y t^2\]

\[2\bar{a}_y (y_f - y_i) = v_{yf}^2 - v_{yi}^2\]